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If T = kinetic energy of entire moving system, and if m represent mass of 1-pound weight, we have

$$T = \frac{m}{2}\Big[9\dot{x}^2 + \dot{x}^2 + (\dot{y}^2 - 2\dot{x}\dot{y} + \dot{x}^2) + 3(\dot{y}^2 + 2\dot{y}\dot{x} + \dot{x}^2) + \tfrac{10}{3}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2 - 2\dot{x}\dot{y}$$

$$+ 2\dot{z}\dot{y} - 2\dot{z}\dot{x}) + 2(\dot{x}^2 + \dot{y}^2 + \dot{z}^2 + 2\dot{x}\dot{z} - 2\dot{y}\dot{z} - 2\dot{x}\dot{y})\Big]$$

$$= \frac{m}{3}(29\dot{x}^2 + 14\dot{y}^2 + 8\dot{z}^2 - 10\dot{x}\dot{y} + 4\dot{y}\dot{z} - 4\dot{x}\dot{z});$$

whence,

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{x}} = \frac{2m}{3}(29\ddot{x} - 5\ddot{y} - 2\ddot{z}) = (9 - 1 - 3 - 1 - 2 - \tfrac{10}{3})mg = -\tfrac{4}{3}mg,$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{y}} = \frac{2m}{3}(14\ddot{y} - 5\ddot{x} + 2\ddot{z}) = (-3 + 2 + 1 + \tfrac{10}{3})mg = \tfrac{10}{3}mg,$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{z}} = \frac{2m}{3}(8\ddot{z} + 2\ddot{y} - 2\ddot{x}) = (-2 + \tfrac{10}{3})mg = \tfrac{4}{3}mg,$$

which give $\ddot{x} = 0$, $\ddot{y} = \frac{1}{3}g$, and $\ddot{z} = \frac{1}{6}g$, and establishes proof sought if we notice that velocity of $3\frac{1}{3}$-pound weight is $\ddot{y} + \ddot{z}$.

Inasmuch as the 9-pound weight always remains at rest if the system starts from rest, the presence of friction in the fixed pulley would not alter the motion of the system as long as no friction be present in the movable pulleys. But if the fixed pulley is frictionless and friction is present in the movable pulleys, then the diminished accelerations of the weights and movable pulleys on the one side of the fixed pulley must, in part, be offset by an upward acceleration of the 9-pound weight on the other side, and in part by the heat of friction generated as long as the friction is not too great to prevent the revolving of the movable pulleys.

352 (Mechanics). Proposed by C. N. SCHMALL, New York City.

A glass rod is balanced partly in and partly out of a cylindrical tumbler, with lower end resting against the vertical wall of the tumbler. If φ and ψ are the maximum and minimum angles, respectively, which the rod can make with the vertical plane, and θ is the angle of friction, show that

$$\theta = \tfrac{1}{2}\tan^{-1}\frac{\sin^3\varphi - \sin^3\psi}{\sin^2\varphi\cos\varphi + \sin^2\psi\cos\psi}.$$

SOLUTION BY WILLIAM HOOVER, Columbus, Ohio.

Let $AB = 2a$ be the rod with its lower end at A; G, its middle point; P, the point of support in the edge of the tumbler; CD, the element of the surface of the tumbler in which A lies; and W = the weight of AB. From P draw a perpendicular to CD cutting it in E, and, similarly, draw GK.

The rod in either extreme position is kept in equilibrium by its weight acting vertically downwards, the two reactions R, S, at A and P respectively, and the friction at A in the direction of the element CD and at P in the direction of the rod.

Resolving vertically and horizontally and taking moments about A,

$$W + S\tan\theta\cdot\cos\varphi = S\sin\varphi - R\tan\theta, \tag{1}$$

$$S\cos\varphi + S\tan\theta\cdot\sin\varphi = R, \tag{2}$$

$$W\cdot GK = S\cdot AP, \tag{3}$$

or

$$W\{(a - x)\sin\varphi + b\} = S\cdot x. \tag{4}$$

From the geometry,

$$x = b/\sin\varphi, \tag{5}$$

and this in (4) gives

$$Wa\sin^2\varphi = b. \tag{6}$$

Substituting R from (2) in (1),

$$W = S\{\sin\varphi(1 - \tan^2\theta) - 2\cos\varphi\tan\theta\}. \tag{7}$$

(6) ÷ (7) gives

$$a\sin^2\varphi = \frac{b/S}{\sin\varphi(1 - \tan^2\theta) - 2\cos\varphi\tan\theta}. \tag{8}$$

For the other extreme angle ψ, $\tan\theta$ changes sign, and we have, by analogy,

$$a\sin^2\psi = \frac{b/S}{\sin\psi(1 - \tan^2\theta) + 2\cos\psi\tan\theta}. \tag{9}$$

(8) ÷ (9) gives

$$\frac{2\tan\theta}{1 - \tan^2\theta} = \tan 2\theta = \frac{\sin^3\varphi - \sin^3\psi}{\sin^2\varphi\cos\varphi + \sin^2\psi\cos\psi}, \tag{10}$$

giving the required angle.

It may be interesting to note that when the upper edge is smooth, the angle of friction is twice as great as that in (10).

Also solved by A. M. HARDING and G. PAASWELL.

353 (Mechanics). Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

A uniform beam of oak, 10 feet long, 15 inches deep and 10 inches wide, sustains, in addition to its own weight, a load of 5,000 lbs. placed at the center. Find the greatest bending moment and the greatest stress in the fibers. Take the specific gravity of oak as 0.934.

SOLUTION BY W. J. THOME, Detroit, Michigan.

Let l, b, d be the length, breadth, and depth, respectively, of the beam; M the greatest bending moment; p, the greatest stress in the fibers; and S, the section-modulus of the beam's cross section. Taking the inch and the pound as units and the weight of a cubic foot of water $= 62.5$ pounds, we have

$$\begin{aligned} M &= \tfrac{1}{4}(5000)l + \tfrac{1}{8}\left(bdl \times 0.934 \times \frac{62.5}{1728}\right)l \\ &= \tfrac{1}{4}(5000)10 \times 12 + \tfrac{1}{8}\left[10 \times 15(10 \times 12) \times 0.934 \times \frac{62.5}{1728}\right]10 \times 12 \\ &= 159121 \text{ inch-pounds.} \end{aligned}$$

$$p = M/S = M/\tfrac{1}{6}bd^2 = 6M/bd^2 = 6 \times 159121/10 \times 15^2 = 424 \text{ lbs. per sq. in.}$$

Also solved by PAUL CAPRON.

354 (Mechanics). Proposed by G. PAASWELL, New York City.

The acceleration of an electric train is constant and equal to a ft. per sec. per sec. Its braking or deceleration is variable and equal to the square root of the velocity. If the distance between stations is 5,000 ft., show that the acceleration must cease and braking ensue when the train is about 960 ft. from the stopping point; also that the maximum velocity attained for a minimum time run is 88 m.p.h. and the time of run is 54 seconds.

SOLUTION BY PAUL CAPRON, U. S. Naval Academy.

Under a constant acceleration, a final speed of 88 mi./hr., or about 129 ft./sec., is acquired, in a distance of 4,040 ft., at the average rate of $64\frac{1}{2}$ ft./sec., in about 62.6 secs. The corresponding acceleration is 2.06 ft./sec.^{2}. Evidently the numerical values are incorrect. It will appear further that the distance to be run under acceleration, the greatest velocity acquired and the time of run are all dependent on the acceleration, and that the time of run is a minimum (about 39.15 secs.) when the acceleration is infinite, its duration zero, and the greatest speed about $261\frac{1}{4}$ mi./hr.

Let the corresponding values of time in seconds, distance in feet, and speed in feet per second, be t, s, v at any point between stations, and let these letters, with appropriate subscripts, represent